UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS 4037/02

Paper 2

October/November 2005

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages and **3** blank pages.

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UNIVERSITY *of* CAMBRIDGE International Examinations

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 Variables V and t are related by the equation

$$V = 1000e^{-kt},$$

where k is a constant. Given that $V = 500$ when $t = 21$, find

(i) the value of k, [2]

(ii) the value of V when $t = 30$. [2]

2 The line $x + y = 10$ meets the curve $y^2 = 2x + 4$ at the points A and B. Find the coordinates of the mid-point of AB. [5]

3 **(i)** Given that $y = 1 + \ln(2x - 3)$, obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Hence find, in terms of p, the approximate value of y when $x = 2 + p$, where p is small. [3]

4 The function f is given by $f : x \mapsto 2 + 5 \sin 3x$ for $0° \leqslant x \leqslant 180°$.

(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

5 The binomial expansion of $(1 + px)^n$, where $n > 0$, in ascending powers of x is

$$1 - 12x + 28p^2x^2 + qx^3 + \ldots .$$

Find the value of n, of p and of q. [6]

6 It is given that $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 5 & p \end{pmatrix}$ and that $\mathbf{A} + \mathbf{A}^{-1} = k\mathbf{I}$, where p and k are constants and $\mathbf{I}$ is the identity matrix. Evaluate p and k. [6]

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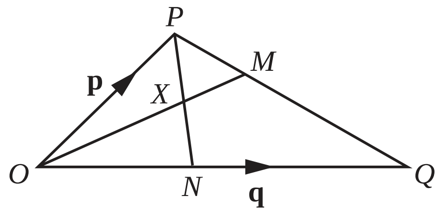


In the diagram $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$, $\overrightarrow{PM} = \frac{1}{3}\overrightarrow{PQ}$ and $\overrightarrow{ON} = \frac{2}{5}\overrightarrow{OQ}$.

(i) Given that $\overrightarrow{OX} = m\overrightarrow{OM}$, express $\overrightarrow{OX}$ in terms of m, $\mathbf{p}$ and $\mathbf{q}$. [2]

(ii) Given that $\overrightarrow{PX} = n\overrightarrow{PN}$, express $\overrightarrow{OX}$ in terms of n, $\mathbf{p}$ and $\mathbf{q}$. [3]

(iii) Hence evaluate m and n. [2]

8 **(a)** Find the value of each of the integers p and q for which $\left(\frac{25}{16}\right)^{-\frac{3}{2}} = 2^p \times 5^q$. [2]

(b) **(i)** Express the equation $4^x - 2^{x+1} = 3$ as a quadratic equation in 2^x. [2]

(ii) Hence find the value of x, correct to 2 decimal places. [3]

9 The function $f(x) = x^3 - 6x^2 + ax + b$, where a and b are constants, is exactly divisible by $x - 3$ and leaves a remainder of -55 when divided by $x + 2$.

(i) Find the value of a and of b. [4]

(ii) Solve the equation $f(x) = 0$. [4]

10 A curve is such that $\frac{d^2y}{dx^2} = 6x - 2$. The gradient of the curve at the point $(2, -9)$ is 3.

(i) Express y in terms of x. [5]

(ii) Show that the gradient of the curve is never less than $-\frac{16}{3}$. [3]

11 **(a)** Each day a newsagent sells copies of 10 different newspapers, one of which is *The Times*. A customer buys 3 different newspapers. Calculate the number of ways the customer can select his newspapers

(i) if there is no restriction, [1]

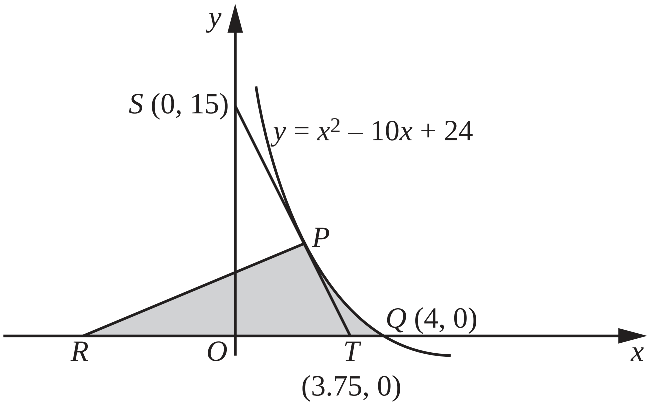
(ii) if 1 of the 3 newspapers is *The Times*. [1]

(b) Calculate the number of different 5-digit numbers which can be formed using the digits 0,1,2,3,4 without repetition and assuming that a number cannot begin with 0. [2]

How many of these 5-digit numbers are even? [4]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram, which is not drawn to scale, shows part of the curve $y = x^2 - 10x + 24$ cutting the x-axis at $Q(4, 0)$. The tangent to the curve at the point P on the curve meets the coordinate axes at $S(0, 15)$ and at $T\,(3.75, 0)$.

(i) Find the coordinates of P. [4]

The normal to the curve at P meets the x-axis at R.

(ii) Find the coordinates of R. [2]

(iii) Calculate the area of the shaded region bounded by the x-axis, the line PR and the curve PQ. [5]

OR

A curve has the equation $y = 2\cos x - \cos 2x$, where $0 < x \leqslant \frac{\pi}{2}$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Given that $\sin 2x$ may be expressed as $2\sin x \cos x$, find the x-coordinate of the stationary point of the curve and determine the nature of this stationary point. [4]

(iii) Evaluate $\int_{\pi/3}^{\pi/2} y \, dx$. [3]

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